Discrete Fourier Transform

Alexandre Trilla, Xavier Sevillano

Departament de Tecnologies Mèdia
LA SALLE – UNIVERSITAT RAMON LLULL
Quatre Camins 2, 08022 Barcelona (Spain)
atrilla@salle.url.edu, xavis@salle.url.edu

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Abstract

This tutorial presents the Fourier transform of use in computational environments: the Discrete Fourier Transform.

With regard to the Discrete-Time Fourier Transform (DTFT) that is defined for sequences of infinite duration, the Discrete Fourier Transform (DFT) is an alternative discrete-time Fourier representation but for finite-duration sequences. The DFT corresponds to a sequence of N samples equally spaced in frequency, of the DTFT of the signal:

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}kn} \tag{1}$$

N usually coincides with the number of samples of $x[n]$. If N is greater than the number of samples of $x[n]$, then $x[n]$ must be padded with zeros.

Regarding the interpretation of the DFT representation of a finite duration sequence, note that a discretisation in the frequency domain (sampling) implies a periodicity in the time domain. The DFT of the finite length sequence corresponds to the Discrete Fourier Series representation of the periodic sequence (where its period equals the finite length sequence).

In addition to the theoretical importance of the DFT as a frequency representation of sequences, the DFT plays a central role in the implementation of a variety of digital signal processing algorithms, such as the Fast Fourier Transform, that is optimised for sequences which length is a power-of-two.